Velocity Ratio , Slip and creep of Belt :

Velocity Ratio ofBelt : It can be defined as the speed of the driven pulley to the speed of the driving pulley.

Let d_1= Dia. Of the driving pulley.

d_2= Dia. Of the driven pulley

N_1=Speed of the driving pulley in rpm.

N_2=Speed of the driven pulley in rpm.

Length of the belt passes over the driving pulley in one minute = $\pi d_1 N_1$

Similarly ,the length of the belt passes over the driven pulley in one minute=$\pi d_2 N_2$

Since the length of the belt passes over the driving pulley in one minute is equal to the length of the belt passes over the driven pulley in one minute, therefore

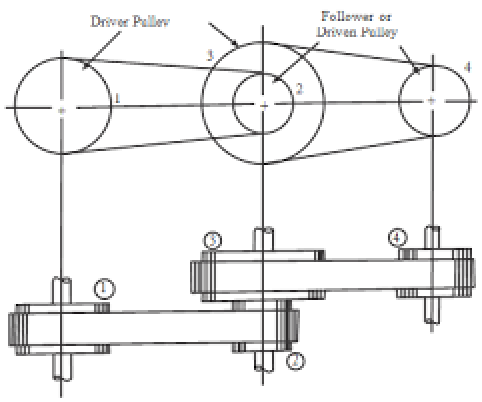
$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\text{Velocity ratio} = \frac{N2}{N1} = \frac{d1}{d2}$$

If the thickness of the belt (t) is considered , then the velocity ratio is given by

$$\text{Velocity ratio,} = \frac{N2}{N1} = \frac{d1+t}{d2+t}$$

Velocity Ratio of compound Belt Drive :



Compound Belt Drive

Let d_1, d_2, d_3 and d_4 = Diameters of pulleys 1 ,2,3 and 4, respectively.

N_1 , N_2, N_3 and N_4 = Speed of the pulleys 1 ,2,3 and 4, respectively .

We know that , velocity ratiosVelocity ratio $=\frac{N2}{N1}=\frac{d1}{d2}$ and $\frac{N4}{N3}=\frac{d3}{d4}$

$$\frac{N2}{N1} \text{X} \frac{N4}{N3} = \frac{d1}{d2} \text{ X} \frac{d3}{d4}$$

$\frac{N4}{N1} = \frac{d1}{d2} \text{x} \frac{d3}{d4}$ ($N_2 = N_3$ as they are connected to the same shaft)

Or $\frac{Speed\ \ of\ \ last\ \ driven\ \ pulley}{Speed\ \ of\ \ first\ \ driver\ \ pulley} = \frac{Product\ \ of\ \ dia.of\ \ driver\ \ pulleys}{Product\ \ of\ \ dia.of\ \ driven\ \ pulley}$

<u>Slip of the Belt</u> : In the belt drive , it is assumed that a firm grip is existing between the belt and pulley , but sometimes , this frictional grip becomes insufficient. This may cause some forward motion of the driver pulley without carrying the belt with it or forward motion of the belt without carrying the driven pulley with it.This phenomenon is known as *slip of the Belt* and it is generally expressed in percentage.

The effect of slip is to decrease the velocity ratio of the system. Therefore , the belt drive should not be used where a definite velocity ratio is very important.

Let , S_1% = % of slip between the driving pulley and the belt.

S_2% = % of slip between the driven pulley and the belt.

Velocity ratio $\frac{N2}{N1}=\frac{d1}{d2}(1-\frac{S}{100})$ --- Where $S=S_1+S_2$ = total percentage of slip.

When the thickness of the belt (t) is considered , then

Velocity ratio, = $\frac{N2}{N1}=\frac{d1+t}{d2+t}(1-\frac{S}{100})$

Creep of the Belt : When the belt moves from the slack side to the tight side to transmit the power, a certain portion of the belt extends and it again contracts when the belt moves from the tight side to the slack side. This is because , the belt material is elastic , it elongates more on tight side than on slack side, as the tension in the tight side is more than the tension in the slack side. Due to these changes of length , there is a relative motion between the belt and pulley surfaces. This relative motion is known as ***creep of belt.***

The total effect of creep is to reduce the speed of the driving pulley or driven pulley .

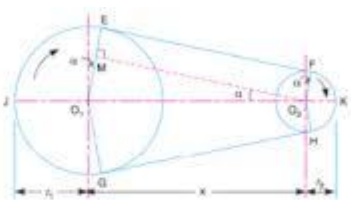
Let, σ_1 and σ_2 = Stresses in the belt on the tight side and slack side respectively.

E= Young's modulus for thebelt material

Then , by considering creep , the velocity ratio is given by,

$$\frac{N2}{N1}=\frac{d1}{d2} \times \frac{E+\sqrt{\sigma 2}}{E+\sqrt{\sigma 1}}$$

Length of the Belt (Open Belt Drive) : An open belt drive is shown in fig , in which both the pulleys rotate in the same direction.



$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

Where,

r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1\,O_2$),

L = Total length of the belt

α = angle of lap

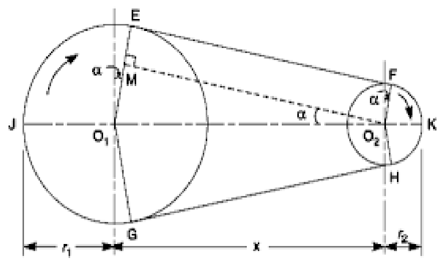
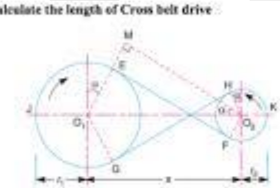


Fig. Length of an open belt drive.

Length of the Belt (Cross Belt Drive) :

Formulae to calculate the length of Cross belt drive



$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

Where,

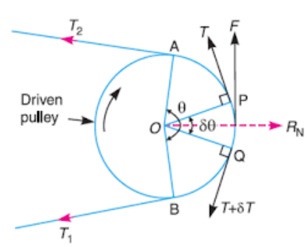
r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1\,O_2$),

L = Total length of the belt

α = angle of lap

Ratio of Driving Tensions :



Ratio of Driving Tensions

Let us consider a driven pulley rotating in the clockwise direction in clockwise direction as shown in fig.

Let T_1 = Tension in the tight side

T_2 = Tension in the slack side

θ= Angle of contact of the belt over the pulley in radians

μ= Coefficient of friction between the belt and the pulley.

Then , $\frac{T1}{T2} = e^{\mu\Theta}$

Or $2.3 \log \left(\frac{T1}{T2}\right) = \mu\theta$

For V- Belt and Rope drives , the ratio of driving tensions is given by ,

$\log_e\left(\frac{T1}{T2}\right) = \frac{\mu\theta}{\sin\beta} = \mu\theta \text{ Cosec } \beta$

where $2\beta =$ Angle of the groove.

Centrifugal Tension : When the belt runs continuously over the pulleys , some centrifugal force is developed due to its own weight , whose effect is to increase the tension on both tight and slack sides.This tension caused by the centrifugal force known as centrifugal tension.

Let , m= mass per unit length of the belt in kg.

T_C= centrifugal tension on tight and slack sides of the belt element in Newton (N)

v= velocity of the belt in mtr/sec.

r= radius of the pulley in mtr.

Then , Centrifugal tension $\mathbf{T_C= mv^2}$

When the centrifugal tension is taken in to account , then total tension in the tight side is

given by, $T_{t1} = T_1 + T_c$

and total tension in slack , $T_{t2} = T_2 + T_c$

Power transmitted $P = (T_1 - T_2)V$ ------ Kw

Initial Tension in the Belt :

When the belt is fitted over the pair of pulleys , the belt ends are joined firmly together, so that it has to move continuously over the pulleys , since the motion of the belt from the driving pulley and driven pulley is governed by the firm grip due to the friction between the belt and pulleys. Inorder to increase this grip , the belt is tightened up. At this stage , even though the pulleys stationary , the belt is subjected to some tension , this tension is known as initial tension.

Let T_0= Initial tension in the belt

T_1= Tension in the tight side of the belt

T_2= Tension in the slack side of the bel ,

α = Coefficient of increase in length of the belt per unit force.

Therefore , $T_0 = \frac{T1+T2}{2}$

Initial tension with consideration of centrifugal tension is given by,

$$T_0 = \frac{T1+T2+2Tc}{2}$$

Where T_c = centrifugal tension.

Maximum Tension in the Belt : The maximum tension in the belt ,

T= Maximum stress x cross sectional area of the belt.

T= σ. b. t

When the centrifugal tension is neglected , then maximum tension in the belt ,

$T = T_1$

When the centrifugal tension is considered ,then

$T = T_1 + T_C$

Power transmitted by the Belt (Flat and V belt) and Rope :

1.The power transmitted by the flat belt is given by :

$P = (T_1 - T_2)\, V$ in N-mtr/sec or watts.

Where , T_1= Tension on tight side in Newton.

T_2= Tension on slack side in Newton.

V = Velocity of the belt in mtr/sec

<u>Maximum Power Transmitted by Belt :</u>

For maximum power transmission , centrifugal tension in the belt must be equal to one third of the maximum allowable belt tension ,

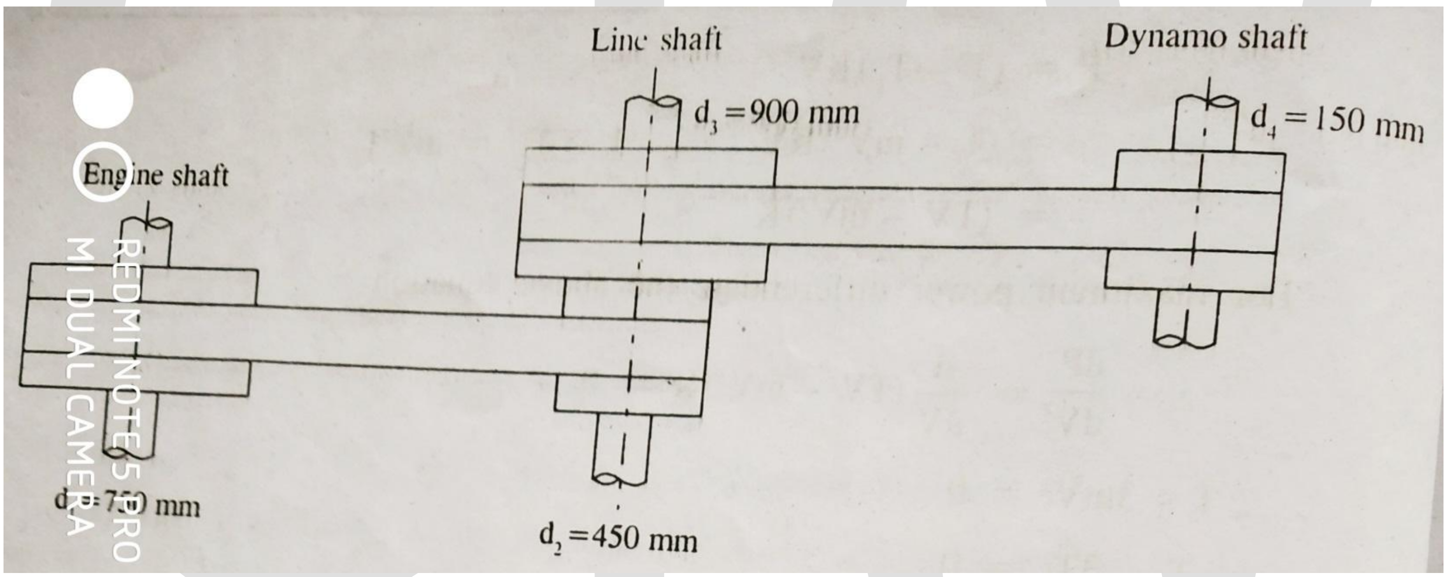
Note : For maximum power , $T_1 = T - T_c = T - \frac{T}{3} = \frac{2}{3} T$

and velocity of the belt : $V_{max} = \sqrt{\frac{T}{3m}}$

Problems on Belt Drive :

1. An engine , running at 150 rpm , drives a line shaft by means of a belt.The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Calculate the speed of the dynamo shaft , when
   i) There is no slip
   ii) There is a slip of 2% at each drive.

Answer : Data given, N_1= 150 rpm ; d_1= 750 mm, d_2=450 mm, d_3= 900mm , d_4=150mm,



1.when there is no slip :$\frac{N4}{N1} = \frac{d1}{d2}$x $\frac{d3}{d4}$

$$\frac{N4}{150} = \frac{750}{450} \text{x} \frac{900}{150} = 10$$

N_4= 150 X10 = 1500 rpm.

2.When there is a slip of 2% at each shaft : $\frac{N4}{N1} = \frac{d1}{d2}$ X$\frac{d3}{d4}$ (1 - $\frac{S1}{100}$) (1 - $\frac{S2}{100}$)

$\frac{N4}{N1}=\frac{d1}{d2}$ X$\frac{d3}{d4}$(1 - $\frac{S1}{100}$) (1 - $\frac{S2}{100}$)

$\frac{N4}{150}=\frac{750}{450}$ X$\frac{900}{150}$(1 - $\frac{2}{100}$) (1 - $\frac{2}{100}$)

$\frac{N4}{150}$ = 9.6, N4 = 9.6 x150 = 1440 rpm .

Problem 2. Calculate the power transmitted by a belt running over a pulley of 600 mm diameter at 200 r.p.m. The coefficient of friction between the belt and the pulley is 0.25, angle of lap is 160^0 and maximum tension in the belt is 2500 N.

Answer : Data: d=600mm = 0.6 m , N = 200 rpm , μ = 0.25,

θ =160^0 =$\frac{160 * \pi}{180}$ = 2.793 radians , T_1 = 2500 N

V = $\frac{\pi * d * N}{60}$ = $\frac{\pi * 0.6 * 200}{60}$ = 6.284 m/sec.

$\log_e (\frac{T1}{T2}) = \mu\theta$

= 0.25 * 2.793 = 0.6982.

$\log_e (\frac{T1}{T2})$ = 0.6982

$(\frac{T1}{T2}) = e^{(0.6982)}$ = 2.01

$T_2 = \frac{T1}{2.01} = \frac{2500}{2.01}$ = 1244 N.

Power transmitted by the belt = P = (T_1 – T_2) V

= (2500 - 1244) * 6.284

= 7890 W or 7.89 kW.

Note: For open belt drive , Sinα = $\frac{r2-r1}{x}$, and Angle of contact, $\theta = (180^0 - 2\alpha)$

For crossed belt drive , Sinα = $\frac{r1+r2}{x}$, and Angle of contact, $\theta = (180^0 + 2\alpha)$

Problem .3.Two pulleys , one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 apart and are connected by a **crossed belt**. Calculate the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when larger pulley rotates at 200 rpm , if the maximum permissible tension in the belt is 1 KN , and the coefficient of friction between the belt and pulley is 0.25.

Answer : Data, d_1=450 mm=0.45 mtr, r_1=0.225 mtr

d_2=200 mm=0.2 mtr, r_2=0.1 mtr , x= 1.95 mtr, N_1= 200 rpm

T_1= 1 KN = 1000 N , $\mu = 0.25$

Velocity of the belt , V = $\frac{\pi * d1 * N1}{60}$ mtr/sec

V = $\frac{\pi * 0.45 * 200}{60}$ = 4.714 mtr/sec.

Length of cross the belt :$L = \pi(r1 + r2) + 2x + \frac{(r1+r2)^2}{x}$

$= \pi(0.225 + 0.1) + 2 * 1.95 + \frac{(0.225+0.1)^2}{1.95}$

= 4.975 mtr.

Angle of contact : **Sinα = $\frac{r1+r2}{x}$ (For crossed belt drive)**

Sinα = $\frac{0.225+0.1}{1.95}$ = 0.1667

$\alpha = \sin^{-1}(0.1667) = 9.6^0$

Therefore ,$\boldsymbol{\theta = (180^0 + 2\alpha)}$ **(For crossed belt drive)**

$\theta = (180^0 + 2 * 9.6) = 199.2^0 = \frac{199.2 * \pi}{180}$ = 3.477 radians.

Power transmitted : $\log_e(\frac{T1}{T2}) = \mu\theta$

$= 0.25 * 3.477 = 0.8692$

Therefore , $(\frac{T1}{T2}) = e^{(0.8692)} = 2.385$

$T_2 = \frac{T1}{2.385} = \frac{1000}{2.385} = 419.28$ N.

Power , $P = (T_1 - T_2)\ V$ ---- Watts

$P = (1000 - 419.28) * 4.714$

$= 2737.5$ Watts . $= 2.737$ kw.

Problem 4. A shaft rotating at 200 rpm. Drives another shaft at 300 rpm and transmits 6 Kw through a belt. The belt is 100 mm wide and 10 mm thick. The distance between the shafts is 4 mtr . The smaller pulley is 0.5 mtr in diameter. Calculate the stress in the belt , if it is open belt drive . Take $\mu = 0.3$.

Answer : $N_1 = 200$ rpm ; $N_2 = 300$ rpm ; P=6 KW =6x1000 = 6000 Watts. b=100 mm; t=10 mm; x = 4 mtr ;

$d_2 = 0.5$ mtr ; $\mu = 0.3$

Velocity of the belt , $V = \frac{\pi * d2 * N2}{60}$ mtr/sec $= \frac{\pi * 0.5 * 300}{60} = 7.855$ mtr/sec

But, $\frac{N2}{N1} = \frac{d1}{d2}$ $d_1 = \frac{N2}{N1} * d_2 = \frac{300}{200} * 0.5 = 0.75$ mtr. $r_1 = 0.375$ mtr

For open belt drive , Sin$\boldsymbol{\alpha} = \frac{\boldsymbol{r2-r1}}{\boldsymbol{x}}$, and Angle of contact, $\boldsymbol{\theta = (180^0 - 2\alpha)}$

but $\sin\alpha = \frac{r1-r2}{x} = \frac{0.375-0.25}{4} = 0.03125$ $\alpha = \sin^{-1}(0.03125) = 1.8^0$

Angle of contact, $\theta = (180^0 - 2\alpha) = 180 - 2 * 1.8 = 176.4^0 = = \frac{176.4 * \pi}{180} = 3.08$ radians.

$\log_e(\frac{T1}{T2}) = \mu\theta$

$\log_e (\frac{T1}{T2}) = \mu\theta = 0.3 * 3.08 = 0.924$

$(\frac{T1}{T2}) = e^{(0.924)} = 2.52$

$T_1 = 2.52 * T_2$ ----------------------------------(1)

Power transmitted , $P = (T_1 - T_2) V$ ---- Watts

$6000 = (2.52 * T_2 - T_2) * 7.855$

$= T_2(2.52 - 1) * 7.855 = 11.9396\ T_2$

$T_2 = \frac{6000}{11.9396} = 502.5$ N

But , $T_1 = 2.52 * 502.5 = 1266.3$ N

Maximum stress in the belt ,$T_1 = \sigma.\ b.\ t$

$1266.3 = \sigma * 100 * 10$

$\sigma = \frac{1266.3}{100*10} = 1.266$ N/mm^2

Problem . 5. A leather belt is required to transmit 7.5 kw from a pulley 1.2 mtr in diameter, running at 250 rpm. The angle embraced is 165^0 and the coffficient of friction between the belt and the pulley is 0.3. If the safe working stress for the leather belt is 1.5 MPa , density of leather 1 Mg/m^3 and thickness of the belt 10 mm, determine the width of the belt taking centrifugal tension in to account.

Answer : Power P= 7.5 kw = 7500 watts , d= 1.2 mtr , N= 250 rpm,

$\theta = 165^0 = \frac{165*\pi}{180} = 2.88$ radians ,$\mu = 0.3$, $\sigma = 1.5$ MPa = 1.5×10^6 N/mm^2

$\rho = 1$ Mg/m^3 = 1×10^6 g/m^3= 1000 kg/m^3 ,

$V = \frac{\mu*d*N}{60} = \frac{\mu*1.2*250}{60} = 15.71$ m/sec

Power transmitted , $P = (T_1 - T_2) V$

$7500 = (T_1 - T_2)\ 15.71$

$T_1 - T_2 = \frac{7500}{15.71} = 477.4$ N -----------------------(1)

$\log_e\left[\frac{T1}{T2}\right] = \mu\,\theta$

$\log_e\left[\frac{T1}{T2}\right] = 0.3 \text{ X } 2.88 = 0.864$

Therefore, $\frac{T1}{T2} = e^{(0.864)} = 2.373$

$T_1 = 2.373 \text{ x } T_2$

Substitute the value of T_1 in equation (1)

$2.373\ T_2 - T_2 = 477.4$

$T_2(2.373 - 1) = 477.4$

$T_2 = \frac{477.4}{1.373} = 347.7$ N

And , $T_1 = 2.373 \text{ x } 347.7 = 825$ N

Mass of the belt / metrelength = Area x Length x Density

$= b \text{ x } t \text{ x } l \text{ x } \rho$

$= b \text{ x } 0.01 \text{ x } 1 \text{ x } 1000 = 10$ b kg.

Centrifugal tension ,$T_c = m\,v^2 = 10 \text{ x } b\ b(15.71)^2$

$= 2468$ b newton

Maximum tension in the belt , $T = \sigma \text{ x } b \text{ x } t = 1.5 \text{ x } 10^6 \text{ x } b \text{ x } 0.01 = 15000$ b Newton

But , $T = T_1 + T_c$

$15000\ b = 825 + 2468\ b$

$(15000 - 2468)\ b = 825$

$b = \frac{825}{12532} = 0.0658$ mtr ---

Problem .6. A rope drive transmits 600 kw from a pulley of effective diameter 4 mtr which runs at a speed of 90 rpm . The angle of lap is 160^0 the angle of groove 45^0 , the coefficient of friction 0.28 , the mass of the rope is 1.5 kg/mtr and the allowable tension in each rope 2400 N. Calculate the number of ropes required.

Answer :Given , P = 600 KW , d = 4 mtr , N = 90 rpm. , $\theta = 160^0 = \frac{160*\pi}{180} =$ 2.8 rad

$2\beta = 45^0$,$\beta = 22.5^0$, $\mu = 0.28$, m =1.5 kg/mtr , T=2400 N

Velocity of rope , $V= \frac{\pi*d*N}{60} = \frac{\pi*4*90}{60}$ = 18.85 m/sec

Centrifugal tension ,$T_c = m\,v^2 = 1.5 \times (18.85)^2$ =533N

When the centrifugal tension is considered ,then

$T= T_1 + T_C$ or $T_1= T - T_C$ = 2400-533 =1867 N

But , $\log_e\left[\frac{T1}{T2}\right] = \frac{\mu\theta}{\sin\beta} = \frac{0.28*2.8}{\sin(22.5)}$ =2.05

Therefore,$\frac{T1}{T2} = e^{(2.05)}$ = 7.768

$T_2 = \frac{1867}{7.768}$ = 240.34 N

Power transmitted per rope , $P = (T_1 - T_2)\,V$ = (1867 -240.34) *18.85 =30662 watts or 30.662 Kw.

Number of ropes =$\frac{Total\ power\ transmitted}{Power\ transmitted\ per\ rope}$ =$\frac{600}{30.662}$ = 19.5 or say 20 ropes

---

Problem 7.: Determine the width of a 9.75 mm thick leather belt required to transmit 15 kW from a motor running at 900 rpm. The diameter of the driving pulley of the motor is 300 mm. The driven pulley runs at 300 rpm and the distance between the centers of two pulleys is 3 meters. The density of the leather is 1000kg/m^3. The maximum allowable stress in the leather is 2.5 MPa . The coefficient of friction between the leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt.

Answer : Given : t=9.75 mm =9.75 x10^{-3}mtr , P = 15kW= 15 x10^3 watts , N_1 = 900 rpm ,

d_1 = 300 mm = 0.3 mtr , N_2 = 300 rpm ,x = 3 mtr , ρ = 1000kg/m^3 ,

σ =2.5 MPa = 2.5 x10^6 N/m^2 , $\mu = 0.3$,

But, $\frac{N2}{N1}=\frac{d1}{d2}$

$d_2 = \frac{N1}{N2} * d1 = \frac{900}{300} * 0.3 = 0.9$ mtr

and , $V = \frac{\pi * d * N}{60} = \frac{\pi * 0.3 * 900}{60} = 14.14$ m/sec

Note: For open belt drive , $\sin\alpha = \frac{r2 - r1}{x}$, and Angle of contact, $\theta = (180^0 - 2\alpha)$

$\sin\alpha = \frac{r2-r1}{x} = \frac{d2-d1}{2x} = \frac{0.9-0.3}{2*3} = 0.1$

$\alpha = \sin^{-1}(0.1) = 5.74^0$

Angle of contact, $\theta = (180^0 - 2\alpha) = 180^0 - 2 * 5.74^0 = 168.52^0$

$= \theta = 168.52^0 = \frac{168.52 * \pi}{180} = 2.94$ radians ,

$\log_e\left[\frac{T1}{T2}\right] = \mu\,\theta = 0.3 \times 2.94 = 0.882$

Therefore, $\frac{T1}{T2} = e^{(0.882)} = 2.42$

$T_1 = 2.42 \times T_2$--------------(1)

Power transmitted , $P = (T_1 - T_2)\,V = 15 \times 10^3 = (2.42T_2 - T_2) \times 14.14$

$= 15 \times 10^3 = T_2\ (2.42-1) \times 14.14$

$= 20.07T_2$

$T_2 = 15 \times 10^3 / 20.07 = 747.4$ N

And $T_1 = 2.42 \times T_2$

$T_1 = 2.42 \times 747.4 = 1808.7$ N

Mass of belt per mtrlength , $m = b \times t \times l \times \rho$

$= b \times 9.75 \times 10^{-3} \times 1 \times 1000$

$= 9.75\ b$ kg

And centrifugal tension , $T_c = m\,v^2 = 9.75 \times b \times (14.14)^2$

$= 1950\ b$ N

Maximum tension in belt , T = σ x b x t

$=2.5 \times 10^6 \times b \times 9.75 \times 10^{-3}$

= 24400 b N

But , T= $T_1 + T_C$

24400 b = 1808.7 + 1950 b

b (24400 – 1950) = 1808.7

$b=\frac{1808.7}{22450}$ = 0.080 mtr = 80 mm

Problem . 8. A pulley is driven by a flat belt , the angle of lap being 120^0. The belt is 100 mm wide by 6 mm thick and density 1000 kg/m^3. If the coefficient of friction is 0.3 and the maximum stress in the belt is not to exceed 2 MPa . Calculate the greatest power which the belt can transmit and the corresponding speed of the belt.

Answer : $\theta = 120^0 = \frac{120*\pi}{180}$ = 2.1 radians , b = 100 mm = 0.1 m , t = 6 mm = 0.006 m ,

ρ = 1000 kg/m^3 , $\mu = 0.3$, σ = 2 MPa = 2 x 10^6 N/ m^2

Maximum tension in the belt , , T= σ x b x t = 2 x 10^6 x 0.1 x 0.006 = 1200 Newton

Mass of the belt per meter length , m = b x t x l x ρ =0.1 x 0.006 x 1 x 1000 = 0.6 kg/min

Speed of the belt for maximum power , $V= \frac{\sqrt{T}}{3\,m} = \frac{\sqrt{1200}}{3*0.6}$ = 25.82m/sec

For maximum power to be transmitted ,$T_c= \frac{T}{3} = \frac{1200}{3}$ = 400 N

Tension on tight side of the belt , $T_1= T - T_c$

= 1200 – 400 = 800 N

Now , $\log_e\left[\frac{T1}{T2}\right]= \mu\,\theta$

=0.3 x 2.1 = 0.63

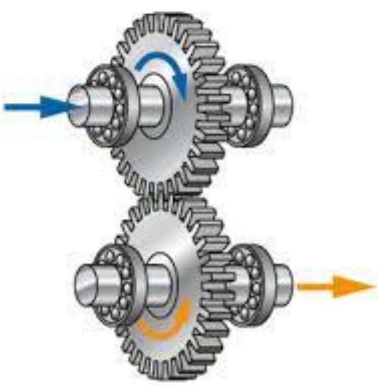
$\frac{T1}{T2} = e^{(0.63)} = 1.88$

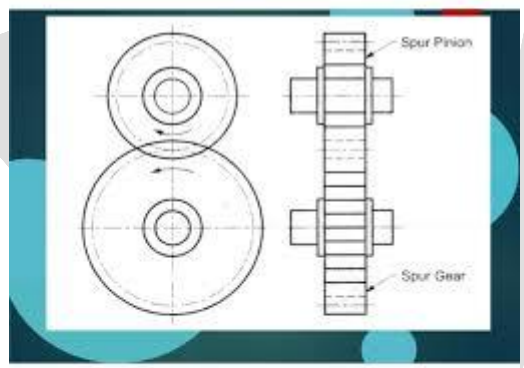
$T_2= \frac{T1}{1.88} = \frac{800}{1.88}$ = 425.5 N

Greatest power which the belt can transmit , P = ($T_1 - T_2$) V = (800 – 425.5) x 25.82 =9670 watts = 9.67 kW

Introduction to Gear Drive : The gears are toothed wheels used for the transmission of motion from one shaft to another or between a shaft and a slide. This is accomplished by a successively enganging the tooth. Gears use no intermediate link or connector and transmit the motion by direct contact.

While transmitting the motion or power between the two shafts, the effect of slipping is to reduce the velocity ratio of the system. In precision machines , where a definite velocity ratio is very essential , in such case the transmission of motion or power is carried out by means of gears. Gear drives are generally used where the centre between the driver and the follower is very small and requires positive drive. A simple gear drive is shown in fig.





Advantages and Disadvantages of Gear Drives :

Advantages ;

1. Positive velocity ratio.
2. Used for transmitting large power.
3. High efficiency.
4. Reliable service.
5. Compact in layout.
6. Transmission of power between shafts of small centre distance.

Disadvantages :

1. Gear manufacturing requires special tools and requirements.
2. Error in tooth cutting create vibrations and noise during operation.
3. High production cost.

Classification of Gears :

According to the position of the shaft axes, there are 3 classification

- Parallel
- Intersection
- Non-parallel & Non-intersecting

Based on the engagement of the gear, there 3 types of gears. they are

- External Gearing
- Internal Gearing
- Rack and pinion

The classification of Gears according to the velocity of gears are 3 types

- Low velocity
- Medium velocity
- High velocity

Based on the tooth profile on the gear surface ,there 3 types of gears. they are

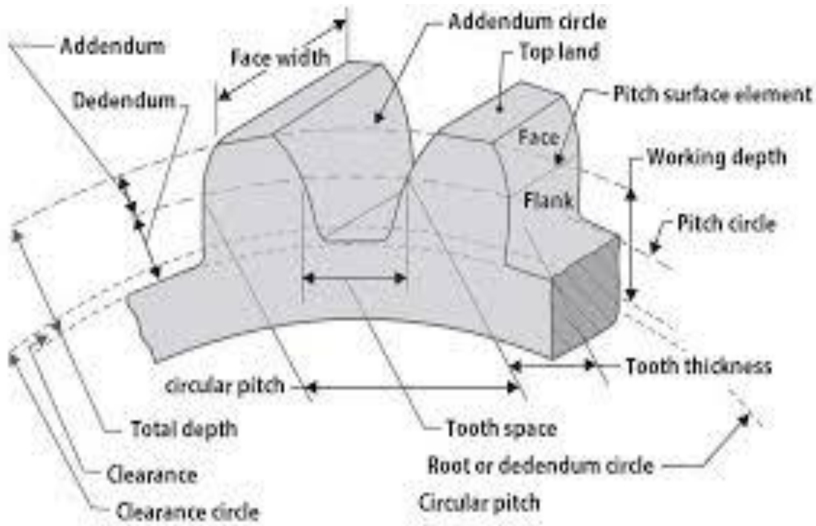
- Gears with straight teeth : Spur gears
- Gears with curved teeth : Helical gears
- Gears with inclined teeth : Spiral gears

Spur Gear Terminology:

Pitch circle: It is an imaginary circle which bypure rolling action , would give the same motion as the actual gear.

Addendum: It is a radial distance of a tooth from the pitch circle to the top of the tooth.

Dedendum: It is a radial distance of a tooth from the pitch circle to the bottom of the tooth.



Spur gear terminology.

Circular pitch : It is the distance measured on the circumference of the pitch circle from the point of one teeth to the corresponding point on the next tooth . It is denoted by 'P_c'.

Pitch circle , $P_c = \frac{\pi d}{T}$

Where , d = Dia . of pitch circle.

T = No. of teeth on the gear.

If two gears are meshing correctly and their diameters are d_1 and d_2 and their teeths are $T_1$and T_2

Then , $P_c = \frac{\pi d1}{T1} = \frac{\pi d2}{T2}$ or $\frac{d1}{d2} = \frac{T1}{T2}$

Diametralpitch : It is the ratio of number of teeth to the pitch circle diameter in mm. It is denoted by 'P_d'

Diametralpitch ,$P_d = \frac{T}{d}$

$= \frac{\pi}{Pc} \left[Pc = \frac{\pi d}{T}\right]$

Where , T = No. of teeth on the gear.

d= Dia . of pitch circle.

Module : It is the ratio of pitch circle diameter in mm to the number of teeth . It is denoted by 'm'

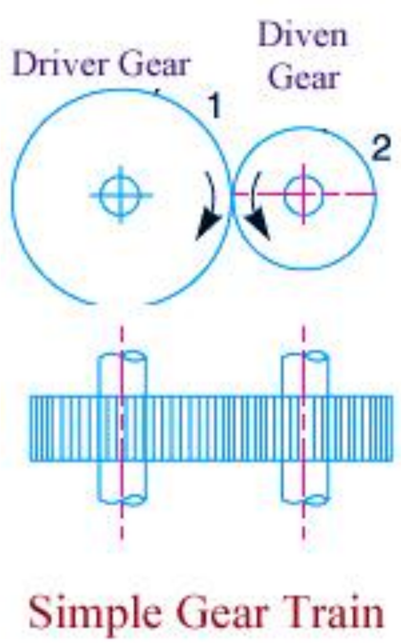
$$\text{Module , m} = \frac{d}{T}$$

Gear Trains : When the two or more gears are made to mesh with each other to transmit power from one shaft to another , then such combination is known as gear train. The type of gear trains used depends upon the velocity ratio required and the relative position of the axis of the shaft.

Types of Gear Trains : Following are the types of gear trains :

1. Simple gear train.
2. Compound gear train.
3. Reverted gear train.
4. Epicyclic gear train.

Simple gear train:



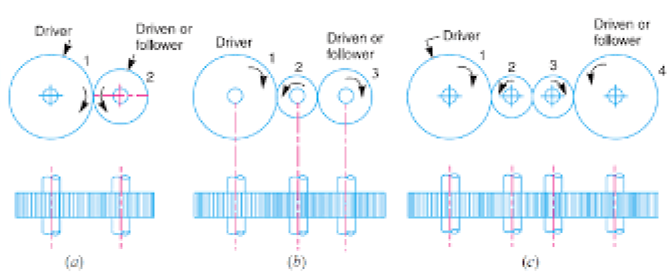


Fig. 4 Simple gear train.

Simple gear train : When there is only one gear on each shaft, as shown in Fig. , it is known as simple gear train. The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. Since the gear 1 drives the gear 2, therefore gear 1 is called the driver and the gear 2 is called the driven or follower. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

Let N1 = Speed of gear 1(or driver) in r.p.m.,

N2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T1 = Number of teeth on gear 1,

andT2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth,

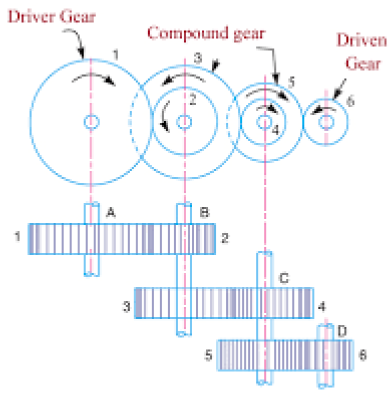
Therefore, Speed ratio $= \frac{N1}{N2} = \frac{T1}{T2}$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as train value of the gear train.

Mathematically, Train value $= \frac{N2}{N1} = \frac{T1}{T2}$

Compound Gear Train : When there are more than one gear on a shaft, as shown in Fig. 13.2, it is called a compound train of gear. We have seen that the idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.



Compound Gear Train

In a compound train of gears, as shown in Fig. the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N1 = Speed of driving gear 1,

T1 = Number of teeth on driving gear 1,

N2 ,N3 ..., N6 = Speed of respective gears in r.p.m.,

and T2 ,T3 ..., T6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

Speed ratio $=\frac{N1}{N2}=\frac{T2}{T1}$ -------------------------(i)

Similarly, for gears 3 and 4, speed ratio is

Speed ratio $=\frac{N3}{N4}=\frac{T4}{T3}$-------------------------(ii)

and ,for gears 5 and 6, speed ratio is

Speed ratio $=\frac{N5}{N6}=\frac{T6}{T5}$-------------------------(iii)

The speed ratio of the compound gear train is given by multiplying the equations (i), (ii) and (iii),

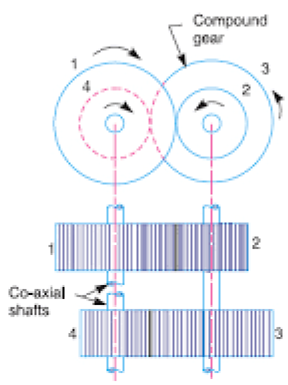
$$\frac{N1}{N2} \text{x} \frac{N3}{N4} \text{x} \frac{N5}{N6} = \frac{T2}{T1} \text{x} \frac{T4}{T3} \text{ x } \frac{T6}{T5} \textbf{ or } \boldsymbol{\frac{N1}{N6} = \frac{T2}{T1} \text{ x } \frac{T4}{T3} \text{ x } \frac{T6}{T5}}$$

Or Speed ratio = $\frac{Speed\ of\ the\ first\ driver}{Speed\ of\ the\ last\ driven}$

Train value = $\frac{Speed\ of\ the\ last\ driven}{Speed\ of\ the\ first\ driver}$

The main advantage of compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.

Reverted Gear Train : When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train as shown in Fig. We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is same.



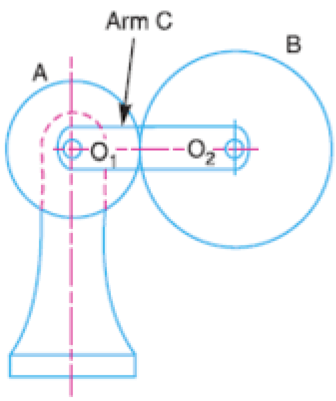
Reverted Gear Train

Then , Speed ratio = $\frac{N1}{N4}$ = = $\frac{T2}{T1}$ x $\frac{T4}{T3}$

Speed ratio = $\frac{Product\ of\ number\ of\ teeth\ on\ drivens}{Product\ of\ number\ of\ teeth\ on\ drivers}$

Epicyclic Gear Train : In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. where a gear A and the arm C have a common axis at O1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2 , about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice- versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate upon and around gear A. Such type of motion is called epicyclic .

Epicyclic gear train.



The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Problems :

1.Two parallel shafts , about 600 mm apart are to be connected by spur gears. One shaft is to run at 360 r.p.m. and the other at 120 r.p.m. Develop the gears , if the circular pitch is to be 25 mm.

Answer : Given : x= 600 mm , N_1 = 360 r.p.m. N_2 = 120 r.p.m. P_c = 25 mm

We know that , V.R. = $\frac{N2}{N1}=\frac{d1}{d2}$, $\frac{d1}{d2}=\frac{120}{360}=\frac{1}{3}$, $d_2 = 3\ d_1$

But , $x=\frac{d1+d2}{2}$, $d_1 + d_2 = 2\ x$ = 2*600=1200 mm ,

Substitute the value of ,' d_2' then $d_1 + 3d_1 = 1200$,

$4d_1 = 1200$,

$d_1 = \frac{1200}{4} = 300$ mm

and ,$d_2 = 3 \times 300 = 90$

$\therefore \quad P_c = \frac{\pi\ d1}{T1}$

$\therefore \quad T_1 = \frac{\pi\ d1}{Pc}$

$T_1 = \frac{\pi * 300}{25} = 37.7 \cong 38$

And ,$\therefore \quad T_2 = \frac{\pi\ d2}{Pc} = \frac{\pi * 900}{25} = 113.1 \cong 114$

Since the values T_1 and T_2 are rounded off to the nearest values , therefore gear diameters have to be recalculated,

$\therefore \quad P_c = \frac{\pi\ d1}{T1}$, $\quad d1 = \frac{Pc \times T1}{\pi}$

$d_1 = \frac{Pc \times T1}{\pi} = \frac{25 \times 38}{\pi} = 302.4$ mm

$d_2 = \frac{Pc \times T2}{\pi} = \frac{25 \times 114}{\pi} = 907.18$ mm

The centredistance , , $\quad x = \frac{d1+d2}{2}$

$x = \frac{302.4+907.18}{2} = 604.79$ mm

Problem .2 Two parallel shafts are to be connected by spur gearing . The approximate distance between the shafts is 600 mm .If one shaft runs at 120 r.p.m and the other at 360 r.p.m. Calculate the number of teeth on each wheel , if the module is 8 mm. Also determine the exact distance apart of the shafts.

Solution : Given : x = 600 mm , N_1 = 120 r.p.m. $\quad N_2$ = 360 r.p.m. $\quad$ m = 8 mm

V.R. = $\frac{N2}{N1} = \frac{d1}{d2}$,

$\frac{d1}{d2} = \frac{360}{120} = 3$

$d_1 = 3d_2$

But , $\quad x = \frac{d1+d2}{2}$, $\quad d_1 + d_2 = 2$ x

$d_1 + d_2 = 2$ x , $= 2 \times 600 = 1200$

$3d_2 + d_2 = 1200$

$4d_2 = 1200$

$d_2 = \frac{1200}{4} = 300$ mm

and $d_1 = 3d_2$

$d_1 = 3 \times 300 = 900$ mm

module , $m = \frac{d1}{T1}$, $T_1 = \frac{d1}{m}$

$T_1 = \frac{900}{8} = 112.5$ say 114

And $m = \frac{d2}{T2}$, $T_2 = \frac{d2}{m}$

$T_2 = \frac{300}{8} = 37.5$ say 38

Since the values T_1 and T_2 are rounded off to the nearest values , therefore gear diameters have to be recalculated,

$\therefore$ $m = \frac{d1}{T1}$

$d_1 = m\, T_1$

$d_1 = 8 \times 114 = 912$ mm

$\therefore$ $m = \frac{d2}{T2}$

$d_2 = m\, T_2$

$d_2 = 8 \times 38 = 304$ mm

The centredistance , , $x = \frac{d1+d2}{2}$

$x = \frac{912+304}{2} = 608$ mm

Problem .3. The number of teeth of a spur gear is 40 and it rotates at 220 rpm . What will be the circular pitch line velocity of it has module of 2 mm ?

Solution : Given: T = 40 , N = 220 R.P.M. , m = 2 mm

∴ $P_c = \frac{\pi\, d}{T}$

$= \pi\, m$

$= \pi \times 2 = 6.283$ mm

We know that , $m = \frac{d}{T}$

d = m x T

d = 2 x 40 = 80 mm = 0.08 mtr

Pitch line velocity , $V = \frac{\pi * d * N}{60} = \frac{\pi * 0.08 * 220}{60} = 0.921$ m/sec